Graphs!

$V = \{a, b, c\}$

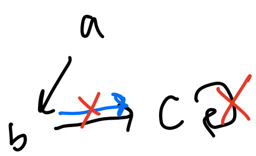
$E = \{(a,b), (b,c)\}$ → directed

Graph = (V, E) → set of edges :

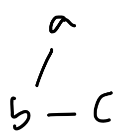
$\{ab, bc\}$

$\{\{a,b\}, \{b,c\}\}$

↑ set of vertices $v \in V$



directed



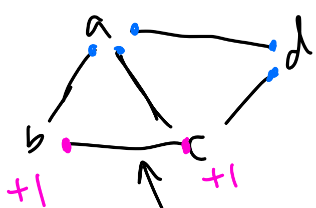
undirected

a & b are neighbors

a & b are adjacent

simple graphs : no self loops, no multiple edges

degree (v) = number of edges that have v as an endpoint



degree (a) = 3, degree (d) = 2

each edge contributes a total of 2 degrees to the entire graph

$$\sum_{v \in V} \deg(v) = 2|E|$$

↓ add all nodes' degrees

↑ cardinality → # of elements in set of edges, or # of edges

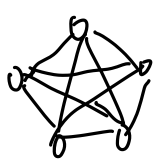
special graphs

$n > 2$

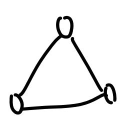
① complete : K_n

n total nodes

K_5 :



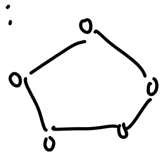
K_3 :



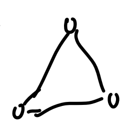
② cycle : C_n

n total nodes

C_5 :



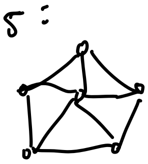
C_3 :



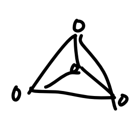
③ wheel : W_n

n+1 total nodes

W_5 :

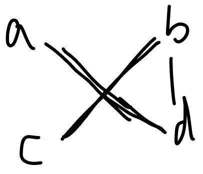


W_3 :



all nodes have deg n-1 | all nodes have deg 2 | all nodes have deg 3

isomorphism



$\equiv$ a — d — b — c
$\cong$ 1 — 2 — 3 — 4

} isomorphic

$f(a) = 1$, $f(d) = 2$, $f(b) = 3$, $f(c) = 4$.

Suppose G_1 and G_2 are graphs, $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$. An isomorphism from G_1 to G_2 is a bijection $f: V_1 \to V_2$ such that edge (a,b) exists iff edge $(f(a), f(b))$ exists. If there is an isomorphism, the graphs are isomorphic.

V_1	V_2
a	1
d	2
b	3
c	4

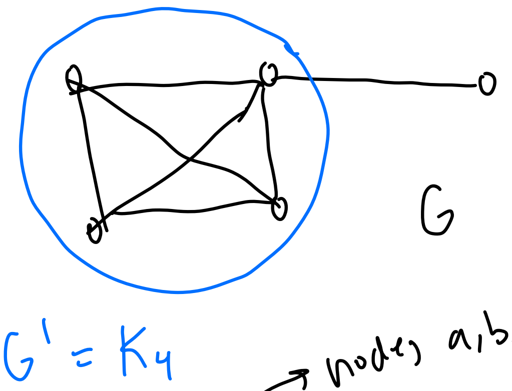
equivalent representation of f

graphs are not isomorphic if:

- different numbers of nodes
- different numbers of edges
- structural differences
  - → number of nodes with degree k
    - ex) G_1 has 2 nodes of deg 5, G_2 has 1 node of deg 5
  - → different subgraphs

subgraphs: $G' = (V', E')$ is a subgraph of $G = (V, E)$ iff

$V' \subseteq V$, $E' \subseteq E$



nodes a, b, c, d.

if G_1 has K_4 as subgraph, and G_2 does not, they are not isomorphic. (State without proof)

how to traverse a graph:

walk: finite sequence of nodes from a to b
- → describe by nodes or edges
- → length is # edges you traverse

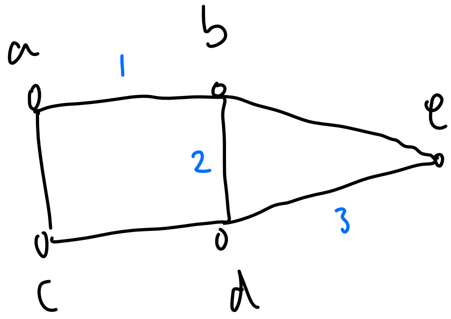
closed: a & b are same node (go back beginning)

open: $a \neq b$

path: no node is repeated

cycle: closed walk in which no other nodes are used more than once (start & end nodes are same)

Euler circuit: closed walk that uses each edge in graph exactly once (nothing about nodes)

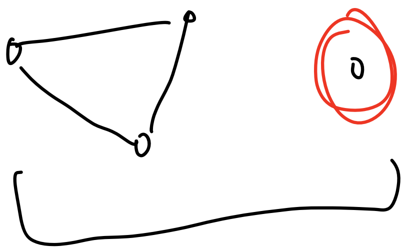


give a length 3 walk from a to e ↳ 3 edges

a, b, d, e

(a,b), (b,d), (d,e)

a graph is connected iff there is a walk between every pair of nodes



G : not connected

2 connected components

distance between 2 nodes is the length of shortest path between them

☆ diameter of graph is largest of all distances between pairs of nodes

look at the textbook for discussion of bipartite graphs